Some Simple Games for Teaching and Research

Part 1: Cooperative Games

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Abstract

Over many years some simple cooperative games have been considered in lectures on game theory. The games were selected in order to provide insight into various normative theories of solution to *n*-person games. It is suggested that the results indicate that when solutions have outcomes in common, predictability is higher than when they are apart. The core is attractive but less so when it is heavily nonsymmetric.

Introduction

From 1961 to 1994 in teaching game theory I have employed a series of different simple games in class or in other lectures to illustrate various points and to heighten student participation[1]. For the most part, the students were not paid and played only once in class. Most of the use of the games was in classes at Yale where the students, for the most part, were undergraduates (seniors) or Master's degree students. Some games were run elsewhere in the United States, in Austria, Australia, Canada, Chile, India and Hong Kong.

Some of the results reported here have been reported elsewhere (Shubik, 1975, 1978, 1986). Many games were used over a semester to illustrate different problems in the game theoretic treatment of cooperative and competitive behavior. In this part the remarks are confined to some cooperative games.

Game Theory, Learning and Psychology

There are many different purposes for which simple experiments can be utilized and many different controls which can be imposed. There is no single satisfactory solution to all *n*-person games. There are many solutions which have been suggested and depending upon both the questions being asked and the context specified, formal game theory has been of considerable use in illuminating various aspects of multi person decision making.

The control of experiments with human decision makers poses many difficult problems. In particular the gap between game theory solutions and what we see when running experiments or eliciting opinions may be attributed to several key factors. In particular intrinsic in much of game theory is the concept of *external symmetry*. This is the

[1]Not reported here are the computerized business games utilized in class as well as the formal experiments involving multistage play of either computer games or matrix games.

assumption that all individuals are equally talented and differ only by the rules specified in the game. Thus, for example, any two individuals are assumed to have the same memory and enculturation unless otherwise specified in the game specification. The psychologists and social psychologists, in contrast concentrate on individual and cultural differences. Another basic source of experimental difficulty is the description of individual preferences, utility functions and payoffs. For the most part the numbers in the payoff functions are either linked to small sums of money or are otherwise merely symbolic abstract numbers to be maximized.

All game experiments contrast with the game theoretic idealizations of intelligent *externally symmetric* individuals with well defined *preferences* and *payoff functions*, each approaching a new game with a *tabula rasa*. Another distinction of considerable importance in gaming is whether or not the individuals play anonymously or whether they play face-to-face. Yet another distinction, of considerable importance is whether the decision-maker is playing to his or her own account or is a fiduciary. In much of actual economic, political and societal decision making, the decision maker is a fiduciary playing with other people's money, rights or lives.

Not only are there problems in measuring individual preferences and utility functions there are many questions to be raised about whether or not individuals make interpersonal comparisons of welfare and whether they are in a position to transfer "utility" in an approximately linear manner. Much of the original von Neumann and Morgenstern development of cooperative theory was based on the considerable simplification that utility was measurable, interpersonally comparable and linearly transferable.

Another difficulty encountered in both socio-psychological and economic experimentation with games is the size of the matrix utilized. Most experimentation has been confined to 2×2 matrix games because of memory, calculation and perception problems (see Miller's classical article, 1956); although if there is considerable regularity in the structure of the matrix larger games have been used, as for example, Fouraker, Shubik and Siegel (1963) utilized matrices around 20×20 for duopoly games.

Three major divisions in game theoretic analysis and in gaming experimentation have been the representation of games in (1) *coalitional form*; (2) *strategic form* and (3) *extensive form*. The first is used extensively in the study of cooperative solution theories and these theories may be considered as primarily normative. The various axiomatizations of the different solution concepts represent different sets of desiderata suggested for an outcome to be considered as a solution.

The *stategic form* has probably been by far the most popular of the experimental game structures used. It frequently comes in two variations. They are a matrix game (usually a 2×2 matrix game) representing a two player game where each agent has a choice between two moves and each player plays once, or a matrix game which is played repeatedly for a fixed or random number of times. In most multistage game experiments the form of presentation is a matrix game which is then played more than once.

There is evidence that the display of the game influences the way individuals play. For example instead of presenting individuals with a 2×2 matrix game to play twice, giving all the results of the first play prior to their second play, one could give all players a strategically equivalent 8×8 matrix to be played once. Shubik, Wolf and Poon (1974) found considerable differences in the way agents played in the two scenarios.

Although the Prisoner's Dilemma , Chicken and The Battle of the Sexes games have been the most popular experimental vehicles there are 78 strategically distinct 2 × 2 matrix games with strict orderings and over 700 2 × 2 strategically different games with ties. In a heroic work Rapoport, Guyer and Gordon (1976) reported experiments with all 78 matrices.

The third representation of the game, the extensive form presents a detailed description suited to dynamics. The extensive form is eminently suited for raising basic questions concerning the concept of threat and the many variations of the basic idea of a noncooperative equilibrium which reflect both learning and teaching (see Van Damme, 1996, for an extensive coverage and Harsanyi and Selten, 1986, for criteria for equilibrium selection and Kohlberg and Mertens, 1986, for a consideration of problems with the extensive form).

The Games And The Motivation For Their Use

The games noted here and in the subsequent parts, were utilized in order to investigate problems in both cooperative and noncooperative game theory and to consider attitudes towards risk and interpersonal comparisons of payoffs. All three major game forms were used in the game experiments noted. This report is devoted to three cooperative games which were investigated in order to explore how the selections of relatively naive agents relate to the various cooperative solutions which have been proposed.

Cooperative Solutions

Games denoted as 1a, 1b and 6 were devoted to eliciting opinions on how to play three cooperative games. The key distinctions among the games are discussed below.

There are many different motivations for running games, with or without monetary rewards; with or without elaborate scenarios and with or without a precise hypotheses which may be falsified by some form of logical or statistical test. The major motivation here was exploratory. The basic question was can we gain some insights into the perceptions and opinions of relatively naive subjects when confronted with more or less one shot strategic problems with little context and history. It is in this spirit that the results are reported without attempt at great generalization.

The three cooperative game solution concepts considered were the core, the value, the nucleolus as well as the price system (which requires an economic story to locate it if the core is large) as well as the symmetric or even split point.

We call a division of the payoff which can be achieved by all agents cooperating, an *imputation* of the game. Intuitively, *the core* is the set of imputations against which no coalition can propose an alternative which they prefer and could achieve if they acted independently. *The value* of a game gives each individual his *a priori* expected combinatoric marginal value considering the worth of an individual's entry into all coalitions. *The nucleolus* is that imputation for which the difference between what it offers and what any coalition can achieve is minimized. *The competitive equilibrium* is obtained by constructing an associated economy which is represented by the game and solving for the efficient price system. The *even split point* can be regarded as an egalitarian solution which ignores the influence or power of subgroups.

Although the prime purpose for utilizing the games was teaching and exploration, I did have a specific conjecture that a point in the core would be most probably selected when the core was large; however, with a one point highly nonsymmetric core the frequency of selection would be considerably attenuated. It would, however be higher if an economic scenario were supplied. I had no clear conjecture for the game (#6) with no core whatsoever. I wished to use the game as a means for gaining some insight into the choices made. In all instances I asked the students to provide a verbal description or rationalization of their choice.

The predominant set of games consisted of plays by nine classes at Yale. There were also data obtained from seven games in India, seven games in Australia and games in Hong Kong elsewhere in the United States.

The information on the gain through cooperation was displayed by means of a characteristic function. The characteristic functions for games 1a, 1b and 6 are shown below.

<u>Game 1a</u>

$$v(A) = v(B) = v(C) = 0$$
$$v(AB) = 100,\ v(AC) = 200,\ v(BC) = 300$$
$$v(ABC) = 400.$$

<u>Game 1b</u>

$$v(A) = v(B) = v(C) = 0$$
$$v(AB) = 0,\ v(AC) = 0,\ v(BC) = 400$$
$$v(ABC) = 400.$$

<u>Game 6</u>

$$v(A) = v(B) = v(C) = 0$$
$$v(AB) = 250,\ v(AC) = 300,\ v(BC) = 350$$
$$v(ABC) = 400.$$

Where A, B and C are the names of the players and v(AC) indicates the amount that players A and C can obtain by collaborating.

Game 1a (with a “fat core” and 1b (with a single point core) are illustrated below:

Figure 1 shows the core and various other solution points for game 1a drawn on a simplex. The core consists of all imputations in the trapezoidal area denoted by ACDE. The value, in this instance lies in the .core at point V and the nucleolus is the centroid of the core denoted by *N*.

Figure 2 shows the game 1b with the one point core. The core, the nucleolus and the competitive equilibrium all coincide, but the value indicated by V is out of the core.

The figure for the third game is not drawn, but we note that the game has an empty core, no competitive equilibrium point and that the value solution is the point (108, 133, 158) and the nucleolus solution is (83, 133, 183).

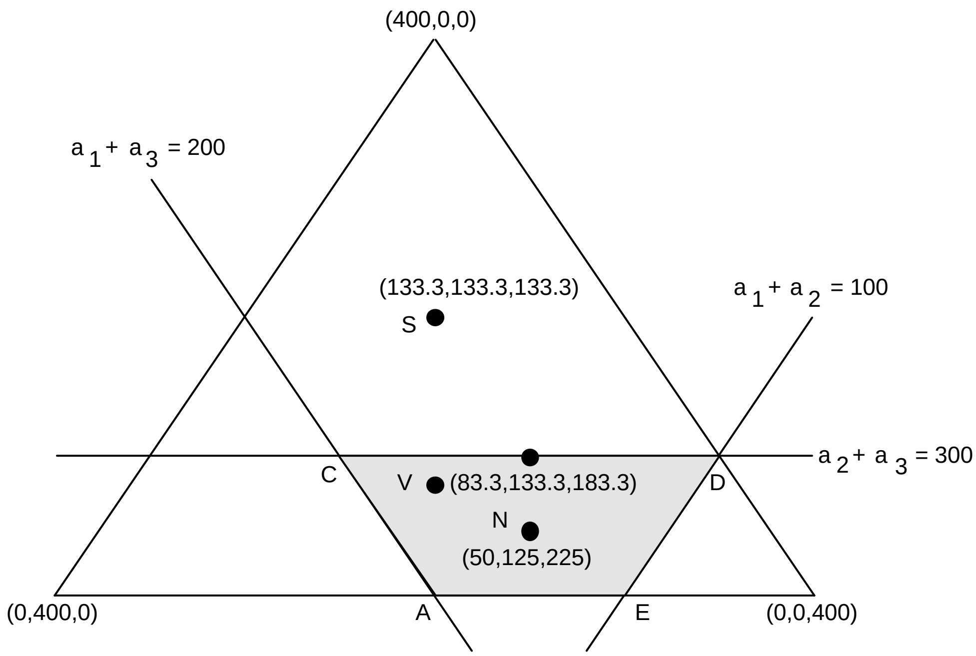

Figure 1

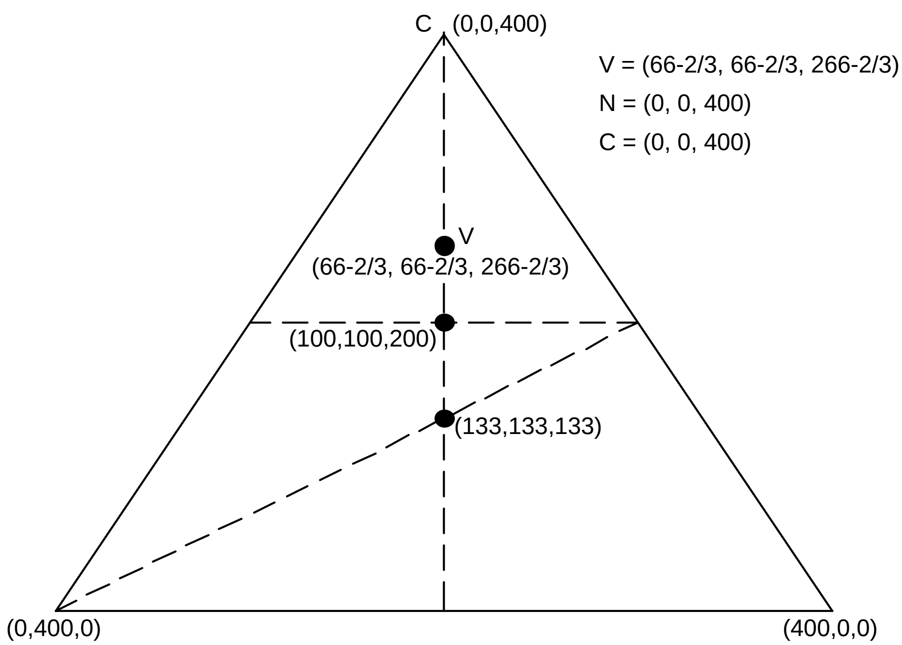

Figure 2

Table 1a illustrates the selections from the Yale Games. We observe that the percentage of imputations selected within the core varies from a low of 86% to a high of 100%. This contrasts with Table 1b for the second game, where the percentage selected in the core varies from a low of 5% to a high of 28.5%.

	Points in the Core									
	Yale 1980	1981	1983	1984	1985	1988	1989	1991S	1991F	Total
(100, 150, 150)	29	20	15	7	19	21	12	15	10	148
(67.5, 133.7, 200)	5	3	7	0	2	2	4	3	7	33
(100, 133, 167)	5	1	2	0		6	4	4	8	36
(100, 125, 175)	2	0	1	2	3	3	1	1	0	13
Other core points	9	9	2	8	14	14	10	10	7	83
TOTAL	50	33	27	17	44	46	31	33	32	313
	Points Not in Core									
(133.3, 133.3, 133.3)	6	4	3	2	3	3	2	0	1	24
Other not in core	2	4	1	0	0	1	1	0	0	6
TOTAL	8	5	4	2	3	4	3	0	1	30
% in core	86%	87%	87%	89%	94%	92%	91%	100%	97%	91%
% not in core	14%	13%	13%	11%	6%	8%	9%	0%	3%	9%
No reply	0	2	1	2	0	0	1	9	3	18
Inefficient/other	1	1	0	2	0	0	1	0	0	5
Illogical	2	0	1	1	0	0	1	1	0	6
TOTAL	3	3	2	5	0	0	3	10	3	29
POPULATION	61	41	33	24	47	50	37	43	36	372

TABLE 1a

	Points in the Core									
	Yale 1980	1981	1983	1984	1985	1988	1989	1991S	1991F	Total
(0, 0, 400)	7	0	3	4	3	2	3	6	4	32
(e, e, 400-2e) e < 1	8	2	4	0	5	3	7	5	4	38
TOTAL	15	2	7	4	8	5	10	11	8	70
	Points Not in Core									
(100, 100, 200)	28	28	16	10	25	24	19	19	18	187
(133.3, 133.3, 133.3)	5	4	3	2	2	4	1	1	0	22
(75, 75, 250)	2	0	0	1	2	1	0	1	0	7
(50, 50, 300)	0	2	3	1	2	5	1	1	2	17
Other core points	3	4	3	3	1	6	4	4	1	29
TOTAL	38	38	25	17	32	40	25	26	21	262
% in core	28%	5%	22%	19%	20%	11%	29%	30%	28%	21%
% not in core	72%	95%	78%	81%	80%	89%	71%	70%	72%	79%
No reply	0	0	0	0	0	0	0	9	5	14
Inefficient/other	1	1	0	2	0	0	1	0	0	5
Illogical	2	0	1	1	0	0	1	1	0	6
TOTAL	3	1	1	3	0	0	2	10	5	25
POPULATION	56	41	33	24	40	45	37	47	34	357

TABLE 1b

We may note that the size of the core in Game 1a relative to the whole simplex is 3/16 = .1875. If choice were random this is the percentage of choices we would expect in the core of game 1a. We would expect essentially no points in the core for game 1b under random choice.

The details for the Australian, Indian and other data are given in Appendix 1. Here a summary is given of the distinctions between the selections in Games 1a and 1b indicating the differences in frequencies of selections of points in the core and the frequencies of the selection of the symmetric point and the top two most frequently selected imputations. The Australian games with the same economics briefings as the Yale, India and other games are included as AusA and the others as AusB.

	% in Core		% at Even		#1		#2	
	1a	1b	1a	1b	1a	1b	1a	1b
Agg	90	27.4	7.6	16.3	29.9	56.3	8	27.4
Yale	91	21	7	6.6	43	56.3	10.7	21
AusA	93	47	5.6	52.9	0	0	11.1	45
AusB	77	37	18	60	9.6	0	9.4	37
India	94	54	4	0	30.1	25.5	4.9	54
Other	76	28.1	18	26.7	39.3	42.3	4.3	28.1

TABLE 2

As might be expected the most frequently selected imputations differed in the two games. The top two choices for game 1a were (100,150,150) 29.9%, then (100,133,167) with 8%.

The top choices for game 1b (100,100,200) with 56.3%, then (0,0,400) with 27.4%. The top choice for 1a was a point in the "equal division core" proposed by Selten (1972). The top choice for game 1b preserved the symmetry of the first two player and could be interpreted naturally in terms of a syndicate approach where the first two players form a syndicate thus converting the game into a symmetric two person game where the first two players split the proceeds from the fair division of the "two person" game. For none of the three games played nine times at Yale was the value selected more than 3% of the time. When the nucleolus, core and competitive equilibrium all coincided they were selected 27.4 % of the time with the other selections indicating a dissatisfaction with the inequity of this outcome. A comparison of top selections is given in Table 5 where the responses of students are contrasted with those of game theorists.

Would or Should?

At one session in Australia the respondents were asked not merely to be the judge for game 1a, but to also answer what they thought would happen if the game were actually played. There were 28 respondents and the differences between "should" and "would" were of note as is shown in Table 3. We observe that (at least among the Australians) when confronted with the "should, would" distinction, the normative prescription was overwhelmingly for equity, whereas the expected behavior was for less symmetry. The two predominant choices for expected behavior were the nucleolus followed by collaboration only among the top two players.

Outcome	Should #	Should %	Would #	Would %
(133,133,133)	21	75	3	10.7
(67,133,200)	1	3.5	2	7.1
(83,133,183)	2	7.1	0	0
(50,125,225)	1	3.5	5	17.9
other in core	3	10.6	7	25
(-,3/2,3/2)	0	0	4	14.3
no reply/error	0	0	7	25

TABLE 3

The Game Without the Core

Game #6 had no core. Time constraints in lecturing were such that outside of Yale I did not have the opportunity to use game 6 and to compare it with the two games with cores. However nine games at Yale provided the basis for the comparison. Table 4 presents the statistics. We observe that 24% of the time the even split was selected, followed by a split of (50,175,175) for 21.2% of the selections, then (125,125,150) for approximately 16%. In all of the games the even split of (133,133,133) is the only solution which does not change position. The core goes from a large area to a point and then disappears; while the value and nucleolus move with the characteristic function's values.

	Yale										
	1980	1981	1983	1984	1985	1988	1989	1991S	1991F	Total	Percentages
(133,133,133)	14	6	2	1	1	2	2	2	2	32	24.24%
(125,125,150)	5	2	3	2	1	2	3	1	2	21	15.91%
(120,133,147)	0	5	0	0	3	1	0	0	0	9	6.82%
(100,133,167)	1	2	0	2	0	1	1	1	0	8	6.06%
100,125,175)	1	1	1	1	1	0	0	2	0	7	5.30%
(89,133,178)	1	1	1	1	1	4	1	0	2	12	9.09%
(75,175,150)	0	0	0	2	0	0	0	3	0	5	3.79%
(67,133,200)	3	0	1	0	1	1	1	3	0	10	7.58%
(50,175,175)	2	4	1	0	4	10	3	3	1	28	21.21%
Others	4	4	15	9	5	15	10	7	10	9	37.44%
TOTAL	31	25	24	18	17	36	21	22	17	211	
No reply	4	0	0	0	4	1	2	6	0	17	
Infeasible	4	1	2	0	1	1	0	0	1	10	
TOTAL	8	1	2	0	5	2	2	6	1	27	
POPULATION	39	26	26	18	22	38	23	28	18	238	

TABLE 4. A Game Without a Core

A Game with a Dummy Player

By accident at the University of West Ontario there was an error in the presentation of game 1b. Instead of the correct characteristic function, player 1 was converted into a dummy, i.e. player 1 made no marginal contribution to productivity. The characteristic function presented was:

<u>Game 1b</u>

$$v(A) = v(B) = v(C) = 0$$
$$v(AB) = 0,\ v(AC) = 0,\ v(BC) = 400$$
$$v(ABC) = 400.$$

Table 5 shows that the dummy is treated as a dummy, except for one small "tip" or charitable contribution. (15 respondents and 2 with no reply excluded from calculation)

Imputation	Percentage
(0, 200, 200)	73.3
(0, 400–x, x)	20
(1, 199.5, 199.5)	6.7

TABLE 5

The Game with Game Theorists

In May 1983 at a conference on game theory I elicited responses from 28 professional game theorists as to how they felt games 1a and 1b should be settled. The responses are shown in Table 6. We note that one responded that he did not believe in cooperative solutions, but would accept the core for game 1a, but did not respond to game 1b. One other offered two solutions for each game and was counted twice. In Table 6 the first two outcomes are the values for games 1a and 1b; the next two are the nucleolus for games 1a and 1b. We see that there is a considerable selection of the Shapley value as the norm whereas the unsophisticate players were not aware of the value or the reasoning behind it.

OUTCOME	G.T. 1a	G.T. 1b	Others 1a	Others 1b
(67.3,133,3,183.3)	53.6	0	3<	3<
(66.6,66.6,266.6)	0	39.3	3<	3<
(50,125,225)	3.6	0	3<	3<
(0,0,400)	0	21.4	0	27.4
(100,100,200)	14.3	14.3	3<	56.3
(100,133,167)	0	0	3<	8
(133.3,133.3,133.3)	3.6	7.1	24	29.9

TABLE 6

The Game with Managers and Renormalization

In the early 1980s games 1a and 1b were used three times with managers of Shell Petroleum. The first two game sessions were precisely as those run at Yale, but the third session employed a different normalization of the characteristic functions. Instead of having the coalition v(123) =400 it was set to 700. The characteristic functions used were:

Game 1a

$$v(A) = v(B) = v(C) = 100$$
$$v(AB) = 300,\ v(AC) = 400,\ v(BC) = 500$$
$$v(ABC) = 700.$$

Game 1b

$$v(A) = v(B) = v(C) = 100$$
$$v(AB) = 200,\ v(AC) = 200,\ v(BC) = 600$$
$$v(ABC) = 700.$$

Theoretically as all that has happened is that each player is rewarded an extra 100. The incremental gain has not been changed. However the perception of asymmetry in the games has been modified. In Table 7 below the imputations for the last session were constructed from the original data by removing 100 from the amount awarded to each. Similarly for Table 8 the imputations in the renormalized games have been reconstructed.

1a	Session 1	Session 2	Renormalized
(100,150,150)	3	6	6
(100,.133,167)	2	4	0
Other in core	10	7	5
(133.3,133.3,133.3)	0	1	0
not in core	1	0	7

TABLE 7

1b	Session 1	Session 2	Renormalized
(100,100,200)	5	9	5
(50,50,300)	1	2	2
(133.3,133.3,133.3)	1	2	0
Other	2	4	6
(0,0,400) and e	1	1	5

TABLE 8

The influence of the renormalization appears to have made (0,0,400), the core for game 1b appear to be less unfair and nonsymmetric. In game 1a the number of points out of the core was notably higher, however as the core is now much smaller relative to the unnormalized simplex it may be more difficult to note intuitively that a point is in the core.

For the first two sessions 94% of the imputations selected were in the core, the average for the Yale games was 91%.

Discussion

The context of the running of these informal experiments was a series of classes and lectures in game theory. There were some differences in briefing which are noted. All but four of the games were run with an economic productivity scenario. (See Appendix 2 for the actual briefing.) Four of the games in Australia were run with a purely abstract presentation of the characteristic function with no productivity story. One game in Australia (noted separately) asked the participants both a "should" and "would" question, i.e., "How do you think the gain should be spit and how to you thing it would be split if the individuals settled among themselves rather than asked for a judgement?"

There were some differences in the make up of the respondents. Those at Yale, in India and in Hong Kong were almost all economics or business students, in Australia there were social science faculty and students and in one instance mathematicians without game theory training.

There is evidence that the story and the briefing can have considerable influence on the play. Professor John Kennedy of Princeton used to half-jokingly remark "you tell me what results you want and give me control of the briefing and I will get you the results." A Ph.D thesis by R. I. Simon (1967) took a two person zero sum game and wrote three scenarios for it. He obtained statistically significant different results for the same game with different scenarios.

Several respondents in Hong Kong and two elsewhere, instead of providing an imputation dividing the 400 into three, divided it into four, giving the judge his cut of the division.

Many of the respondents provided justification or explanation of their selections. I have not made a formal analysis of the written material, although it is available. In essence, the written responses varied encompassing considerations of fairness, equity, symmetry, productivity, efficiency and coalitional power. But these desiderata may clash with each other. In particular this was illustrated in game 1b where the nonsymmetry of the core drove many respondents to selecting points out of the core. When the core was fat, as in game 1a, almost all of the respondents chose points in the core. The reason for the high percentage of selection of the point (100,150,150) was the it is the point in the core of game 1a which while still in the core gets as close as possible to the even split solution.

The accidental use of a characteristic function with player 1 as a dummy instead of as in game 1b gave support to the standard "dummy axiom" in game theory that a dummy should receive no gain. Even here however one respondent left a little for the dummy.

The corporate respondents were much like the economics students, except when the game was renormalized.

There was a wide discrepancy between the professional game theorists and the other respondents. But the game theorists were not unanimous on normative choices. One was antipathetic to cooperative game theory. However the predominant view was that the

Shapley value provided the best *a priori* single point normative solution, with the next most favored imputation in Game 1a being (100,100,200), the most symmetric point in the equal treatment core. It is my belief that it is extremely difficult to perform highly controlled experiments involving socio-economic behavior with humans except in highly context relevant situations. The respondent's mind is not a *tabula rasa* and an apparently controlled sterile environment in the laboratory may magnify the lack of control over the influence of the individual's history.

It is my belief that an alternative or supplementary approach to the laboratory is to gather large samples from teaching uses and from ongoing activities. Experimental subjects are expensive, however cheap large, less controlled samples may provide useful insight. There is still much to be learned from activities such as chess or casino gambling. If the boards at all chess clubs were sensitized to gather and automatically data process the games played and some of games at Las Vegas and other casinos were redesigned for automated data bank construction large samples could be gathered at near to zero marginal cost.

If games were more utilized in educational procedures it would be feasible to automate the type of material discussed here so that the large number of observations would serve as a source of insight into both normative and actual competitive choice behavior. The games discussed here were relatively cheap to run in both time and money and were of educational use. There are undoubtedly some games in which the paying of the players is important. There are others in which it is probably not so.

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APPENDIX 1
Games 1a and 1b Australia

	Australia 1	2	(math) 3	Economics Briefing 4	5	6	Total
			Points in the Core				
(100,150,150)	6	0	2	0	0	0	8
(67.3,133.7,200)	5	3	7	0	2	2	19
(100,133,167)	5	1	2	0	6	6	20
(100,125,175)	2	0	1	2	3	3	11
Other core points	9	9	2	8	14	14	56
TOTAL	27	13	14	10	25	25	114
			Points Not in Core				
(133.3,133.3,133.3)	6	4	3	2	3	3	21
Other not in core	2	1	1	0	0	1	5
TOTAL	8	5	4	2	3	4	26
% in core	77%	72%	78%	83%	89%	86%	81%
% not in core	23%	28%	22%	17%	11%	14%	19%
No reply	0	2	1	2	0	0	5
Inefficient/other	1	1	0	2	0	0	4
Illogical	2	0	1	1	0	0	4
TOTAL	3	3	2	5	0	0	13
POPULATION	38	21	20	17	28	29	153

GAME 1a

	Australia 1	2	(math) 3	Economics Briefing 4	5	6	Total
			Points in Core				
(0,0,400)	6	0	2	0	0	0	8
(e,e,400-2e) e<1	2	1	2	0	6	2	13
TOTAL	8	1	4	0	6	2	21
			Points Not in Core				
(100,100,200)	0	0	0	0	0	0	0
(133.3,133.3,133.3)	8	3	7	3	5	4	30
(75,75,250)	0	0	0	0	0	0	0
(50,50,300)	0	0	0	0	0	0	0
Other core points	1	0	0	0	0	0	1
TOTAL	9	3	7	3	5	4	31
% in core	47%	25%	36%	0%	55%	33%	40%
% not in core	53%	75%	64%	100%	45%	67%	60%
No reply	0	0	0	0	0	0	0
Inefficient/other	1	1	0	2	0	0	4
Illogical	2	0	1	1	0	0	4
TOTAL	3	1	1	3	0	0	8
POPULATION	20	5	12	6	11	6	60

GAME 1b

Games 1a and 1b India

	1	2	3	4	5	6	7	Total	
				Points in the Core					
(100,150,150)	4	5	2	7	8	5	0	31	17.51%
(67.3,133.7,200)	1	7	0	4	3	1	1	17	9.60%
(100,133,167)	0	0	1	4	0	0	0	5	2.82%
(100,125,175)	0	0	0	0	0	0	0	0	0.00%
Other core points	2	15	5	21	35	26	9	113	63.84%
TOTAL	7	27	8	36	46	32	10	166	93.79%
				Points Not in Core					
(133.3,133.3,133.3)	1	0	1	1	1	3	0	7	3.95%
Other not in core	0	1	0	1	1	1	0	4	2.26%
TOTAL	1	1	1	2	2	4	0	11	
% in core	88%	96%	89%	95%	96%	89%	100%	94%	
% not in core	13%	4%	11%	5%	4%	11%	0%	6%	
No reply	11	22	0	2	11	5	2	53	
Inefficient/other	0	0	0	2	0	0	0	2	
Illogical	0	0	1	1	0	0	0	2	
TOTAL	11	22	1	5	11	5	2	57	
POPULATION	19	50	10	43	59	41	12	234	

GAME 1a

	1	2	3	4	5	6	7	Total	
				Points in Core					
(0,0,400)	0	0	0	0	23	5	2	30	54.55%
(e,e,400-2e) e<1	0	0	0	0	0	0	0	0	0.00%
TOTAL	0	0	0	0	23	5	2	30	54.55%
				Points Not in Core					
(100,100,200)	0	0	0	0	8	2	4	14	25.45%
(133.3,133.3,133.3)	0	0	0	0	0	0	0	0	0.00%
(75,75,250)	0	0	0	0	0	0	0	0	0.00%
(50,50,300)	0	0	0	0	5	0	1	6	10.91%
Other core points	0	0	0	0	9	4	2	15	27.27%
TOTAL	0	0	0	0	14	4	7	25	45.45%
% in core	0%	0%	0%	0%	62%	56%	22%	55%	
% not in core	0%	0%	0%	0%	38%	44%	78%	45%	
No reply	0	0	0	0	14	30	3	47	
Inefficient/other	0	0	0	0	8	2	0	10	
Illogical	0	0	0	0	0	0	0	0	
TOTAL	0	0	0	0	22	32	3	57	
POPULATION	0	0	0	0	59	41	42	112	

GAME 1b

Other Games

	Bdlr	SF1	g1	g2	HK	W Ont	Shell1	Shell2	Shell3
				Points in the Core					
(100,150,150)	7	3	8	3	12	4	3	6	6
(67.3,133.7,200)	0	1	0	0	1	0	0	0	0
(100,133,167)	1	0	0	1	2	0	2	4	0
(100,125,175)	0	0	0	0	0	0	1	0	0
Other core points	2	6	4	5	4	7	10	7	5
TOTAL	10	10	12	9	19	11	16	17	11
				Points Not in Core					
(133.3,133.3,133.3)	6	1	6	1	3	1	0	1	0
Other not in core	3	0	0	0	2	0	1	0	7
TOTAL	9	1	6	1	5	1	1	1	7
% in core	53%	91%	67%	90%	79%	92%	94%	94%	61%
% not in core	47%	9%	33%	10%	21%	8%	6%	6%	39%
No reply	0	1	10	1	9	9	1	0	0
Inefficient/other	1	1	0	0	2	2	0	0	0
Illogical	0	0	0	2	1	1	0	0	0
TOTAL	1	2	10	3	12	12	1	0	0
POPULATION	20	13	28	13	36	24	18	18	18

GAME 1a

	Bdlr	SF1	g1	g2	HK	W Ont	Shell1	Shell2	Shell3
				Points in Core					
(0,0,400)	6	2	2	2	1		1	1	4
(e,e,400-2e) e<1	2	1	2	1	1		0	0	1
TOTAL	8	3	4	3	2	error	1	1	5
				Points Not in Core					
(100,100,200)	0	6	8	6	10		5	9	5
(133.3,133.3,133.3)	8	0	4	0	1		1	2	0
(75,75,250)	0	0	0	0	0		0	0	0
(50,50,300)	0	1	0	1	1		1	2	2
Other core points	1	1	0	1	2		2	4	6
TOTAL	9	8	12	8	14		9	17	13
% in core	47%	27%	25%	275	13%				
% not in core	53%	73%	75%	73%	88%				
No reply	3	2	12	2	20		8	0	0
Inefficient/other	1	0	0	0	0		0	0	0
Illogical	2	0	0	0	0		0	0	0
TOTAL	6	2	12	2	20		8	0	0
POPULATION	23	13	28	13	36	error	18	18	121

GAME 1b

APPENDIX 2

GAME 1

a. Three individuals can, by cooperation on a job earn \$400 altogether; if A and B cooperate they earn \$100, if A and C cooperate they earn \$200, if B and C cooperate they earn \$300. The man left out earns nothing. This information can be summarized as follows:

$$V(A) = V(B) = V(C) = 0$$
$$V(AB) = 100,\ V(AC) = 200,\ V(BC) = 300$$
$$V(ABC) = 400$$

You are the judge, the three tell you that they have decided to work together to earn \$400, but they cannot decide how to split the income. Write down three a_1, a_2, a_3 such that:

$$a_1 + a_2 + a_3 = 400$$

This is your split of the income, give your reasons for choosing it.

b. Suppose that instead of the above productivity C was vital to the job and only one of A or B were really needed. The earnings are summarized as follows:

$$V(A) = V(B) = V(C) = 0$$
$$V(AB) = 0,\ V(AC) = 400,\ V(BC) = 400$$
$$V(ABC) = 400$$

State how you would split the \$400 here and give you reasons.

GAME 6

You originally acted as judge in the division of proceeds between 3 individuals with different productivity who had agreed to cooperate. You were asked to act as judge once more in a somewhat different case. The values of the coalitions are given below:

$$V(A) = V(B) = V(C) = 0$$
$$V(AB) = 250,\ V(AC) = 300,\ V(BC) = 350$$
$$V(ABC) = 400$$